Lösungsvorschläge 26.04.2010

Integration :: Teil 3

HINWEIS:
1. ohne Gewähr, Probe selbst (differenzieren)
2. Umformungen nachvollziehen (!!), nicht fragen

LÖSUNGSVORSCHLÄGE

$\int [f(t)]^{n} f'(t)\,dt$

① $F(x) := \int_0^x \frac{t}{\sqrt{1+t^2}}\,dt = \int_1^{\sqrt{1+x^2}} \cancel{\frac{t}{r}} \cdot \cancel{\frac{r\,dr}{t}}$

$r := \sqrt{1+t^2} \qquad \frac{dr}{dt} = \frac{1}{\cancel{2}} \frac{\cancel{2}t}{\sqrt{1+t^2}} = \frac{t}{r}$

$\Rightarrow \quad dt = \frac{r\,dr}{t}$

$= \int_1^{\sqrt{1+x^2}} 1\,dr = r \Big|_1^{\sqrt{1+x^2}} = \sqrt{1+x^2} - 1$

② $F(x) := \int_0^x \frac{\sqrt{t}}{(a+\sqrt{t^3})^2}\,dt = \frac{2}{3}\int_a^{a+\sqrt{x^3}} \frac{1}{r^2}\,dr$

$r := a + \sqrt{t^3} \qquad \frac{dr}{dt} = \frac{3}{2}\sqrt{t} \quad \Leftrightarrow \quad dt = \frac{2}{3}\frac{dr}{\sqrt{t}}$

$= \frac{2}{3}\left[-\frac{1}{r}\right]_a^{a+\sqrt{x^3}} = \frac{2}{3}\left(\frac{1}{a} - \frac{1}{a+\sqrt{x^3}}\right)$

③ $F(x) := \int_a^x \frac{\left(\frac{a}{t}+2b\right)^2}{t^2}\,dt = \int_{1+2b}^{\frac{a}{x}+2b} -\frac{r^2}{a}\,dr$

$r := \frac{a}{t} + 2b \qquad \frac{dr}{dt} = -a\frac{1}{t^2} \qquad dt = -\frac{t^2}{a}\cdot dr$

$$= -\frac{1}{a}\left[\frac{1}{3}\tau^3\right]_{1+2b}^{\frac{a}{x}+2b} = \frac{1}{3a}\left[(1+2b)^3 - \left(\frac{a}{x}+2b\right)^3\right]$$

④ $$F(x) := \int_0^x \left[t^2 \cdot \sqrt[3]{t^3-1}\right] dt$$

$$\boxed{\tau := t^3 - 1 \qquad \frac{d\tau}{dt} = 3t^2 \iff dt = \frac{d\tau}{3t^2}}$$

$$= \frac{1}{3}\int_{-1}^{x^3-1} \tau^{\frac{1}{3}}\, d\tau = \cancel{\frac{1}{3}}\left[\cancel{\frac{3}{4}}\tau^{\frac{4}{3}}\right]_{-1}^{x^3-1}$$

$$= \frac{1}{4}\left((x^3-1)^{\frac{4}{3}} - (-1)^{\frac{4}{3}}\right)$$

⑤ $$F(x) := \int_0^x \frac{\tan^3(t)}{\cos^2(t)}\, dt = \int_0^{\tan(x)} \tau^3\, d\tau$$

$$\boxed{\tau := \tan(t) \qquad \frac{d\tau}{dt} = \frac{1}{\cos^2(t)} \qquad dt = \cos^2(t)\, d\tau}$$

$$= \frac{1}{4}\tau^4 \Big|_0^{\tan(x)} = \frac{1}{4}\tan^4(x)$$

⑥ $$F(x) := \int_0^x \left[\sqrt[7]{e^t + 13} \cdot e^t\right] dt$$

$$\boxed{\tau = e^t + 13 \qquad \frac{d\tau}{dt} = e^t \qquad dt = \frac{d\tau}{e^t}}$$

$$= \int_{14}^{e^x+13} \sqrt[7]{r}\, dr = \int_{14}^{e^x+13} r^{\frac{1}{7}}\, dr$$

$$= \frac{7}{8} r^{\frac{8}{7}} \Big|_{14}^{e^x+13} = \frac{7}{8} \cdot \left(\left(e^x+13\right)^{\frac{8}{7}} - 14^{\frac{8}{7}} \right)$$