Übungsbogen 8, A10 (KW47) – Lösungsvorschlag

a)

$$S \neq 0 \Rightarrow u \neq 0$$

$$u = \frac{S}{1 + \alpha S - \alpha S^2} \iff \frac{1}{u} = \frac{1 + \alpha S - \alpha S^2}{S}$$

$$\iff \alpha S^2 - \alpha S - 1 = -\frac{S}{u} \iff \alpha S^2 + S\left(\frac{1}{u} - \alpha\right) = 1$$

$$\iff S^2 + S\left(\frac{1}{\alpha u} - 1\right) + \left[\frac{1}{2}\left(\frac{1}{\alpha u} - 1\right)\right]^2 = \frac{1}{\alpha} + \frac{1}{4}\left(\frac{1}{\alpha u} - 1\right)^2$$

$$\iff \left[S + \frac{1}{2}\left(\frac{1}{\alpha u} - 1\right)\right]^2 = \frac{1}{\alpha} + \frac{1}{(2\alpha u)^2} - \frac{1}{2\alpha u} + \frac{1}{4}$$

$$= \frac{4\alpha u^2}{(2\alpha u)^2} + \frac{1}{(2\alpha u)^2} - \frac{2\alpha u}{(2\alpha u)^2} + \frac{\alpha^2 u^2}{(2\alpha u)^2}$$

$$= \frac{1}{(2\alpha u)^2} \cdot \left(u^2(\alpha^2 + 4\alpha) - u 2\alpha + 1\right)$$

$$\iff S = \frac{1}{2} - \frac{1}{2\alpha u} + \frac{1}{2\alpha u} \cdot \sqrt{u^2(\alpha^2 + 4\alpha) - u 2\alpha + 1}$$

$$= \frac{\alpha u - 1 + \sqrt{u^2(\alpha^2 + 4\alpha) - u 2\alpha + 1}}{2\alpha u}$$

- Check 1 $(\alpha = u = 1) \Rightarrow S = 1$

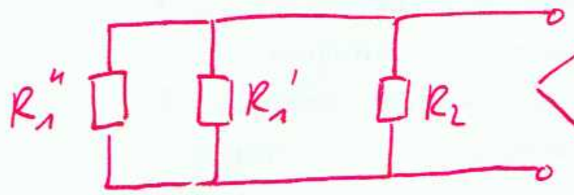
$$\frac{1 - 1 + \sqrt{1 \cdot 5 - 2 + 1}}{2} = \underline{\underline{1}}$$

- Check 2 $\left(\alpha = 1,\ u = \frac{2}{5}\right)$

$$\frac{5}{4} \cdot \left(\frac{2}{5} - 1 + \sqrt{\cancel{\frac{4}{25} \cdot 5 - \frac{4}{5}} + 1}\right) = \underline{\underline{\frac{1}{2}}}$$

b) Bestimmung von R_i

↳ Spgsqu ≙ Kurzschluss



$$\boxed{R_i := R_2 \| R_1' \| R_1''}$$

$$R_i = \frac{1}{\frac{1}{R_2} + \frac{1}{R_1'} + \frac{1}{R_1 - R_1'}} \cdot \frac{R_1}{R_1}$$

$$= \frac{1}{\alpha + \underbrace{\frac{1}{S} + \frac{1}{1-S}}_{\frac{1 \cancel{-S} \cancel{+S}}{S\cdot(1-S)}}} R_1 \cdot \frac{S\cdot(1-S)}{S\cdot(1-S)}$$

$$= \frac{S\cdot(1-S)}{1+\alpha S(1-S)} R_1$$

c) $$I_K = \frac{U_S}{R_i} = \frac{\left(\frac{S}{1+\alpha S(1-S)}\right)\cdot U_q}{\left(\frac{S\cdot(1-S)}{1+\alpha S(1-S)}\right)\cdot R_1} = \frac{U_q}{(1-S)R_1}$$

$$= \frac{U_q}{R_1 - R_1 \cdot \frac{R_1'}{R_1}} = \frac{U_q}{R_1 - R_1'} = \frac{U_q}{R_1''} \quad ✓$$